FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

S.No.	
R.No.	

APPLIED MATH, PAPER-I

TIME ALLOWED: 3 HOURS **MAXIMUM MARKS:100**

NOTE: **(i)** Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION – A** and **THREE** question from **SECTION – B**. All questions carry **EQUAL** marks.
(ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) Show that in orthogonal coordinates: **(5+5)**

(i) $\nabla \times (A_1 e_1) = \frac{e_2}{h_1 h_3}\frac{\partial}{\partial u_3}(A_1 h_1) - \frac{e_3}{h_1 h_2}\frac{\partial}{\partial u_2}(A_1 h_1)$,

(ii) $\nabla \bullet (A_1 e_1) = \frac{1}{h_1 h_2 h_3}\frac{\partial}{\partial u_1}(A_1 h_2 h_3)$.

(b) Write Laplace's equation in parabolic cylindrical coordinates. **(10)**

Q.2. (a) Evaluate $\iint_S \vec{A} \bullet \hat{n}\, ds$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2\hat{k}$ and s is the surface of cylinder $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$. **(10)**

(b) Verify Green's theorem in the plane for

$$\oint_c (xy + y^2)dx + x^2 dy$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

Q.3. (a) Find centre of mass of a right circular solid cone of height h. **(10)**

(b) A light thin rod, 4m long, can turn in a vertical plane about one of its point which is attached to a pivot. If weights of *3kg and 4kg* are suspended from its ends it rests in a horizontal position. Find the position of the pivot and its reaction on the rod. **(10)**

SECTION – B

Q.4. (a) Find the radial and transverse components of the velocity of a particle moving along the curve $ax^2+by^2=1$ at any time t if the polar angle $\theta = ct^2$. **(10)**

(b) A particle is projected vertically upwards. After a time t, another particle is sent up from the same point with the same velocity and meets the first a height h during the downward flight of the first. Find the velocity of projection. **(10)**

Q.5. (a) If a point P moves with a velocity v given by

$$v^2=n^2 (ax^2+bx+c),$$

show that P executes a simple harmonic motion. Find also, the centre, the amplitude and the time period of the motion. **(10)**

(b) A particle of mass m moves on xy-plane under the force **(10)**

$$\vec{F} = -\frac{k}{r^4}\vec{r},$$

where r is its distance from the origin O. If it starts on the positive x-axis at a distance "a" from O with speed v_o in a direction making an angle θ with the positive x-direction, prove that at time t,

$$\ddot{r} = \frac{ma^2 v_o^2 \sin^2\theta - k}{mr^3}$$

Q.6. (a) Define angular momentum and prove that rate of change of angular momentum of a particle about a point O is equal to the tarque (about O) of the force acting on the particle. **(10)**

(b) Find the least speed with which a particle must be projected so that it passes through two points P and Q at heights h_1 and h_2, respectively. **(10)**

Q.7. (a) Discuss the polar form of an orbit and prove that when a particle moves under central force, the areal velocity is constant. **(10)**

(b) A particle moves under a central repulsive force μ/r^3 and is projected from an apse at a distance "a" with velocity V. Show that the equation to the path is **(10)**

$$r \cos p\,\theta = a$$

and that the angle θ described in time t is

$$\frac{1}{p}\tan^{-1}\frac{pVt}{a},$$

where $$p^2 = 1 + \frac{\mu}{a^2 v^2}, \qquad \mu = GM.$$

Q.8. (a) Define the terms moment of inertia and product of inertia, and find the moment of inertia of uniform solid sphere of mass m and radius "a". **(10)**

(b) Let AB and BC be two equal similar rods freely hinged at B and lie in a straight line on the smooth table. The end A is struck by a blow P perpendicular to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

S.No.	
R.No.	

APPLIED MATH, PAPER-II

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION–A, ONE** question from **SECTION–B** and **TWO** questions from **SECTION–C**. All questions carry **EQUAL** marks. **(ii) Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. (a) Using method of variation of parameters, find the general solution of the differential equation.

$$y'' - 2y' + y = \frac{e^x}{x} . \qquad \textbf{(10)}$$

(b) Find the recurrence formula for the power series solution around $x=0$ for the differential equation

$$y'' + xy = e^{x+1}. \qquad \textbf{(10)}$$

Q.2. (a) Find the solution of the problem **(10)**

$$u'' + 6u' + 9u = 0$$
$$u(0) = 2, \;\; u'(0) = 0$$

(b) Find the integral curve of the equation

$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = -(x^2 + y^2). \qquad \textbf{(10)}$$

Q.3. (a) Using method of separation of variables, solve **(10)**

$$\frac{\partial^2 u}{\partial t^2} = 900\frac{\partial^2 u}{\partial x^2} \qquad \begin{cases} 0 < x < 2 \\ t > 0 \end{cases} ,$$

subject to the conditions

$$u(0,t) = u(2,t) = 0$$
$$u(x,0) = 0 \qquad \frac{\partial u}{\partial t}\Big|_{t=0} = 30\sin 4\pi x.$$

(b) Find the solution of **(10)**

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 4e^{3y} + \cos x.$$

SECTION – B

Q.4. (a) Define alternating symbol $\in_{ijk}$ and Kronecker delta δ_{ij}. Also prove that **(10)**

$$\in_{ijk} \in_{lmk} = \delta_{il}\,\delta_{jm} - \delta_{im}\,\delta_{jl} .$$

(b) Using the tensor notation, prove that

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \bullet \vec{B}) - \vec{B}(\nabla \bullet \vec{A}) + (\vec{B} \bullet \nabla)\vec{A} - (\vec{A} \bullet \nabla)\vec{B} \qquad \textbf{(10)}$$

Q.5. (a) Show that the transformation matrix

$$\mathbf{T} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

is orthogonal and right-handed. **(10)**

(b) Prove that **(10)**

$$l_{ik}\ l_{jk} = \delta_{ij}$$

where l_{ik} is the cosine of the angle between *ith-axis* of the system K' and *jth-axis* of the system *K*.

SECTION – C

Q.6. (a) Use Newton's method to find the solution accurate to within 10^{-4} for the equation **(10)**

$$x^3 - 2x^2 - 5 = 0, \qquad [1, 4].$$

(b) Solve the following system of equations, using Gauss-Siedal iteration method **(10)**

$$4x_1 - x_2 + x_3 = 8,$$
$$2x_1 + 5x_2 + 2x_3 = 3,$$
$$x_1 + 2x_2 + 4x_3 = 11.$$

Q.7. (a) Approximate the following integral, using Simpson's $\frac{1}{3}$ rules **(10)**

$$\int_0^1 x^2 e^{-x} dx.$$

(b) Approximate the following integral, using Trapezoidal rule **(10)**

$$\int_0^{\pi/4} e^{3x} \sin 2x\, dx.$$

Q.8. (a) The polynomial **(10)**

$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

has one real zero in [-1, 0]. Attempt approximate this zero to within 10^{-6}, using the Regula Falsi method.

(b) Using Lagrange interpolation, approximate. **(10)**

f(1.15), if f(1) = 1.684370, f(1.1) = 1.949477, f(1.2) = 2.199796, f(1.3) = 2.439189, f(1.4) = 2.670324

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BPS-17 UNDER
THE FEDERAL GOVERNMENT, 2010

Roll Number

APPLIED MATH, PAPER-I

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	**(i)** Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION – A** and **THREE** question from **SECTION – B**. All questions carry **EQUAL** marks. **(ii) Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. Explain the following giving examples and supported by figures: **(5+5+5+5)**

(a) Gradient
(b) Divergence
(c) Curl
(d) Curvilinear Coordinates

Q.2. Given that A,B,C are vectors having components along axis. Prove that: **(10+10)**

(a)

$$B \times C = \begin{vmatrix} i & j & k \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

(b) $A \times B \times C = A_x B_x C_x (i \times k) + A_y B_x C_y (j \times k)$

Q.3. (a) State and prove Stokes Theorem **(10)**

(b) Given that V=4y i+x j + 2z k, find
$\int(\square \times V).\ nd\sigma$ over the hemi sphere $x^2+y^2+z^2=a^2$, z>=0. **(10)**

SECTION – B

Q.4. Discuss the following systems supported by figures/diagrams:

(a)
- Equilibrium of a System coplanar forces **(5)**
- Centre of mass of right circular solid cone of height h. **(5)**

(b) Centre of gravity of a rigid body of any shape. **(10)**

Q.5. (a) What is Simple Harmonic Motion? Discuss it in detail using Derivatives with respect time. **(10)**

(b) Describe the Simple Harmonic Motion of a pendulum and Calculate the time period of the motion. **(10)**

Q.6. (a) Derive expression for the following:
- Moment of inertia **(5)**
- Product of inertia **(5)**

(b) Calculate the moment of inertia of solid sphere of mass m=37 and radius a=15.
Derive the general expression. **(10)**

Q.7. (a) Explain Kepler's Laws. **(10)**

(b) What is Impulsive Motion? Derive its equation. **(10)**

Q.8. (a) Define Work, Torque, Power and energy. **(10)**

(b) A cricket ball is thrown vertically upwards, it attained the maximum height h after t Seconds. Calculate its. **(10)**
- Velocity of projection in direction vertically upward.
- Acceleration when it returns to the point of projection.

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BPS-17 UNDER
THE FEDERAL GOVERNMENT, 2010

Roll Number

APPLIED MATH, PAPER-II

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION–A, ONE** question from **SECTION–B** and **TWO** questions from **SECTION–C**. All questions carry **EQUAL** marks. **(ii) Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. Solve the following equations:

(a) $d^2y/dx^2 + 5\, dy/dx + 6y = x$ **(10)**

(b) $d^2y/dx^2 + 5\, y\, x = e^x$ **(10)**

Q.2. (a) Derive Cauchy Rieman partial differential equations. **(10)**

(b) Derive Lapace Equation. **(10)**

Q.3. Solve:

(a) $(\partial^2/\partial x^2 + \partial^2/\partial x\partial y + \partial^2/\partial y^2)\, u = 4\, e^{3y}$ **(10)**

(b) u" + 6u' + 9=0; Given that u(0)=2 and u'(0)=0. **(10)**

SECTION – B

Q.4. (a) Discuss the following supported by examples:

- Tensor, **(5)**
- $\in_{ijk} \in_{lmk}$ **(5)**
- Scaler Fields for a continuously differentiable function f=f(x,y,z) **(5)**

(b) Can we call a vector as Tensor, discuss.
What is difference between a vector and a tensor?
What happens if we permute the subscripts of a tensor? **(5)**

Q.5. (a) Discuss the simplest and efficient method of finding the inverse of a square matrix a_{ij} of order 3x3. **(10)**

(b) Apply any efficient method to compute the inverse of the following matrix A: **(10)**

$$\mathbf{A} = \begin{bmatrix} 25 & 2 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

SECTION – C

Q.6. (a) Develop Gauss Siedal iterative Method for solving a linear system of equations A x = b, where A is the coefficient matrix. **(10)**

(b) Apply Gauss Siedal iterative Method to solve the following equations: **(10)**

$$\begin{aligned} 25X_1 + 2X_2 + X_3 &= 69 \\ 2X_1 + 10X_2 + X_3 &= 63 \\ X_1 + 2X_2 + X_3 &= 43 \end{aligned}$$

Q.7. (a) Derive Simpson's Rule for finding out the integral of a function f(x) from limits x=a to x=b for n=6 subintervals (i.e. steps). **(10)**

(b) Apply Simpson's Rule for n=6 to evaluate: **(10)**

$$\int_0^1 f(x)dx \quad \text{where} \quad f(x) = 1/(1 + x2).$$

Q.8. (a) Derive Lagrange Interpolation Formula for 4 points: **(10)**

(b) A curve passes through the following points:
(0,1),(1,2),(2,5),(3,10). Apply this Lagrange Formula to interpolate the polynomial. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.
(ii) **Use of Scientific Calculator is allowed.**
(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Find the divergence and curl $\vec{f}$ If $\vec{f} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + (x^2y + 3z^2)\hat{k}$ **(10)**

(b) Also find a function φ such that $\nabla\varphi = \vec{f}$ **(10)**

Q.2. (a) Find the volume $\iint_R xy\,dA$ where R is the region bounded by the line y = x – 1 and the parabola $y^2 = 2x + 6$. **(10)**

(b) Evaluate the following line intergral: **(10)**

$\int_C y^2dx + xdy$ *where* $c = c_2$ is the line segment joining the points (-5, -3) to (0, 2), and c = c_2 is the arc of the parabola x = 4 - y ^{2}.

Q.3. (a) Three forces P, Q and R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant is **(10)**

$\sqrt{p^2 + Q^2 + R^2 - 2QR\cos A - 2RP\cos B - 2PQ\cos C}$

(b) A hemispherical shell rests on a rough inclined plane whose angle of friction is λ. Show that the inclination of the plane base to the horizontal cannot be greater than arcsin(**2 sin** λ) **(10)**

Q.4. (a) A uniform square lamina of side $2a$ rests in a vertical plane with two of its sides in contact with two smooth pegs distant b apart and in the same horizontal line. Show that if $\frac{\theta}{\sqrt{2}} < b < a$, a non symmetric position of equilibrium is possible in which $b(\sin\theta + \cos\theta) = a$ **(10)**

(b) Find the centre of mass of a semi circular lamina of radius a whose density varies as the square of the distance from the centre. **(10)**

Q.5. (a) Evaluate the integral $\int_0^1 \int_{x^2}^{x} (x^2 + y^2) dy dx$ **(10)**

also show that the order of integration is immaterial.

(b) Find the directional derivative of the function at the point P along z – axis **(10)**
$f(x, y) = 4xz^3 - 3x^2 y^2 z, P = (2,-1,2)$

SECTION – B

Q.6. (a) A particle is moving along the parabola $x^2 = 4ay$ with constant speed v. Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is $\sqrt{5a}$ **(10)**

(b) Find the distance travelled and the velocity attained by a particle moving in a straight line at any time t, if it starts from rest at t =0 and is subject to an acceleration $t^2 + \sin t + e^t$ **(10)**

Q.7. (a) A particle moves in the xy – plane under the influence of a force field which is parallel to the axis of y and varies as the distance from x – axis. Show that, if the force is repulsive, the path of the particle supposed not straight and then **(10)**

$$y = a \cosh nx + a \sinh nx$$

where a and b are constants.

(b) Discuss the motion of a particle moving in a straight line with an acceleration x^3, where x is the distance of the particle from a fixed point O on the line, if it starts at t = 0 from a point x = c with the velocity $c^2/\sqrt{2}$. **(10)**

Q.8. (a) A battleship is steaming ahead with speed V and a gun is mounted on the battleship so as to point straight backwards and is set at angle of elevation α. If v_0 is the speed of projection (relative to the gun) show that the range is $\frac{2v_0}{g} \sin\alpha (v_0 \cos\alpha - V)$ **(10)**

(b) Show that the law of force towards the pole of a particle describing the survey $r^n = a^n \cos n\theta$ is given by $f = \frac{(n+1)h^2 a^{2n}}{r^{2n+3}}$ where h is a constant. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.

(ii) **Use of Scientific Calculator is allowed.**

(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Solve by method of variation of parameter **(10)**

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x 1nx$$

(b) Solve first order non-linear differential equation **(10)**

$$x\frac{dy}{dx} + y = y^2 1nx$$

Q.2. (a) Solve **(10)**

$$c^2 u_{xx} = u_{tt}\,.$$

$$u(0,t) = 0$$

$$u(l,t) = 0$$

$$u(x,0) = \lambda\, Sin\left(\frac{\pi}{l}x\right)$$

$$u_t(x,0) = 0$$

(b) Solve $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$ **(10)**

Q.3. (a) Work out the two dimensional metric tensor for the coordinates p and q given by **(10)**

$$p = (xy)^{\frac{1}{3}},\ q = \left(x^2 / y\right)^{\frac{1}{3}}$$

(b) Prove that $\Gamma^d_{ab} = \frac{1}{2}g^{dc}\left(g_{ac,b} + g_{bc,a} - g_{ab,c}\right)$ **(10)**

Q.4. (a) Work out the Christoffel symbols for the following metric tensor **(10)**

$$g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

(b) Work out the covariant derivative of the tensor with components **(10)**

$$\begin{pmatrix} r\cos\theta & ar\sin\varphi & ar \\ \sin\theta\sin\varphi & a\sin\theta\cos\varphi & a \\ \cos\varphi & a\sin\varphi & 0 \end{pmatrix}$$

Q.5. (a) Find recurrence relations and power series solution of $(x-3)y' + 2y = 0$ **(10)**

(b) Solve the Cauchy Euler's equation $x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1$ **(10)**

(10)

SECTION – B

Q.6. (a) Find the positive solution of the following equation by Newton Raphson method **(10)**

$$2 \sin x = x$$

(b) Solve the following system by Jacobi method: **(10)**

$$10x_1 - 8x_2 = -6$$
$$-8x_1 + 10x_2 - x_3 = 9$$
$$-x_2 + 10x_3 = 28$$

Q.7. (a) Evaluate the following by using the trapezoidal rule. **(10)**

$$\int_0^1 (x+1)dx$$

(b) Evaluate the following integral by using Simpson's rule **(10)**

$$\int_0^4 e^x \, dx$$

Q.8. (a) Solve the following equation by regular falsi method: **(10)**

$$2x^3 + x - 2 = 0$$

(b) Calculate the Lagrange interpolating polynomial using the following table: **(10)**

x	0	1	2
f(x)	1	0	-1

also calculate f (0.5).

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2012

Roll Number

APPLIED MATHS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
(ii) Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A** and **ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C. ALL** questions carry **EQUAL** marks.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) **Use of Scientific Calculator is allowed.**

SECTION-A

Q. 1. Solve the following differential equations:

(a) $y''' - 3y'' + 2y' = \dfrac{e^x}{1+e^{-x}}$ **(10)**

(b) $y' = \dfrac{2xye^{(x/y)^2}}{y^2 + y^2e^{(x/y)^2} + 2x^2e^{(x/y)^2}}$ **(10)**

Q. 2. (a) Find the series solution of the following differential equation:
$y'' - xy = 0$ **(10)**

(b) Use the method of Fourier integrals to find the solution of initial value problem with the partial differential equation.

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad ; \quad (-\infty < x < \infty)$$

And with initial condition $u(x,0) = f(x)$ **(10)**

Q. 3. (a) Solve $x^2y'' - 3xy' + 5y = x^2 \sin(\ln x)$ **(10)**

(b) Find the solution of wave equation

$\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ with boundary and initial conditions

$u(0,t) = u(l,t) = 0, \quad u(x,0) = f(x), \quad \dfrac{\partial u(x,t)}{\partial t} = g(x)$ **(10)**

SECTION-B

Q. 4. Discuss the following terms: **(5x4=20)**

(i) Tensors
(ii) Kronecker delta
(iii) Contraction
(iv) Metric Tensor
(v) Contravariant tensor of order two

Q. 5. (a) Prove that $\left\{ \begin{matrix} i \\ ij \end{matrix} \right\} = \dfrac{\partial}{\partial x^i}(\log \sqrt{g})$ **(10)**

(b) Prove that $\Delta = \begin{vmatrix} \delta_{m1} & \delta_{m2} & \delta_{m3} \\ \delta_{n1} & \delta_{n2} & \delta_{n3} \\ \delta_{p1} & \delta_{p2} & \delta_{p3} \end{vmatrix} = \in_{mnp}$ and $\in_{ijk}\in_{mnp} = \begin{vmatrix} \delta_{mi} & \delta_{mj} & \delta_{mk} \\ \delta_{ni} & \delta_{nj} & \delta_{nk} \\ \delta_{pi} & \delta_{pj} & \delta_{pk} \end{vmatrix}$

Hence prove that $\in_{ijk}\in_{mnp} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$ **(10)**

APPLIED MATHS, PAPER-II

SECTION-C

Q. 6. (a) (i) What is the difference between secant and false position method? Show also graphically. **(5+5=10)**

(ii) Prove that $x_{n+1} = x_n - \frac{f(x_n)}{f^2(x_n)}$

(b) Solve the following system by Jacobi method. (Up to four decimal places).

$8x + y - z = 8$

$2x + y + 9z = 12$

$x - 8y + 12z = 35$

Q. 7. (a) Evaluate by $\frac{3}{8}$ Simpson's rule **(10)**

$\int_0^3 x\sqrt{1+x^2}\,dx$; with n = 6

Also calculate the absolute error.

(b) The amount *A* of a substance remaining in a reacting system after an interval of time *t* in a certain chemical experiment is given by following data:

A:	94.8	87.9	81.3	68.7
t:	2	5	8	14

Find *t* when *A*=80. **(10)**

Q. 8. (a) If $f(x) = x^3$, show that $f(a,b,c) = a + b + c$ **(10)**

(b) Solve by trapezoidal rule

$\int_0^{2\pi} x \sin x\,dx$; with $n = 8$ **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:
- **(i)** Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper**.
- **(ii)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B.** All questions carry **EQUAL** marks.
- **(iii)** **Use of Calculator is allowed.**
- **(iv)** Extra attempt of any question or any part of the attempted question will not be considered.

SECTION-A

Q.1. **(a)** Find a function φ such that $\nabla\varphi = \overset{\leftrightarrow}{f}$ **(10)**

$$\overset{\leftrightarrow}{f} = x\hat{\imath} + 2y\hat{\jmath} + 2\hat{k}$$

(b) Prove that **(10)**

$$\nabla\varphi^n = n\varphi^{n-1}\nabla\varphi$$

Q.2. **(a)** Show that for any vectors $\vec{a}$ and $\vec{b}$ **(10)**

$$\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 2\left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2\right)$$

(b) Prove that **(10)**

$$\left(\vec{a}\times\vec{b}\right)\bullet\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right) = \left(\vec{a}\bullet\vec{b}\times\vec{c}\right)^2$$

Q.3. **(a)** The greatest result that two forces can have is of magnitude P and the least is of magnitude Q. Show That when they act an angle α their resultant is of magnitude **(10)**

$$\sqrt{P^2\cos^2\alpha/2 + Q^2\sin^2\alpha/2}$$

(b) A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the rod is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$ **(10)**

Q.4. **(a)** Three forces P, Q and R act along the BC, CA and AB respectively of triangle ABC. Prove that if $P\cos A + Q\cos B + R\cos C = 0$, then the line of action of the resultant passes through the circum center of the triangle. **(10)**

(b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1}\left(\frac{wa}{(W+w)(a+l)}\right)$$

Q.5. **(a)** Find the volume $\iint_R \left(x^3 + 4y\right)\, dA$ where R is the region bounded by the parabola $y = x^2$ and the line $y = 2x$. **(10)**

(b) Evaluate the following line integral **(10)**

$$\int_c x^2\, dy$$

bonded by the triangle having the vertices (-1,0) to (2,0), and (1,1)

SECTION-B

Q.6. **(a)** The position of a particle moving along an ellipse is given by $\overleftrightarrow{r} = a\cos t\,\hat{i} + b\sin t\hat{j}$. If $a > b$, find the position of the particle where its velocity has maximum or minimum magnitude. **(10)**

(b) Prove that the speed at any point of a central orbit is given by: **(10)**

$$vp = h,$$

When h is the areal speed and p is the perpendicular distance from the centre of force, of the tangent at the point, Find the expression for v when a particle subject to the inverse square law of force describes an ellipse, a parabolic and hyperbolic orbit.

Q.7. **(a)** A particle is moving with the uniform speed v along the curve **(10)**

$$x^2 y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$$

Show that its acceleration has the maximum value at $\dfrac{10v^2}{9a}$

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a, whose centre is a height h vertically above a point O of the ground. If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O, show that Y satisfies the equations **(10)**

$$KY^2 + Y\left(a^2 - 2hK\right) + K\left(h^2 - a^2\right) = 0,$$

where $K = h + \dfrac{ga^2}{2v^2{}_0}$

Q.8. **(a)** Find the tangential and normal components of the acceleration of a particle describing the ellipse **(10)**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

With uniform speed v when the particle is a $a > b$

(b) Find the velocity acquired by a block of wood of mass M lb., which is free to recoil when it is struck by a bullet of mass m lb. moving with velocity v in a direction passing through the centre of gravity. If the bullet is embedded a ft., show that the resistance of the wood to the bullet, supposed uniform, is $\dfrac{Mm^2}{2(M+m)ga}$ lb.wt. and that the time of penetration is $\dfrac{2a}{v}$ sec., during which time the block will move $\dfrac{ma}{m+M}$ ft. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE:
- **(i)** Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
- **(ii)** Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A** and **ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C ALL** questions carry **EQUAL** marks**.**
- **(iii)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(iv)** **Use of Calculator is allowed.**

SECTION-A

Q.No.1. Solve the following equations:

(a) $\dfrac{d^3y}{dx^3}+\dfrac{dy}{dx}=Sec^2x$ **(10)**

(b) $\dfrac{2dy}{dx}-\dfrac{x}{y}+x^3Cos\ y=0$ **(10)**

Q.No.2. **(a)** Find the power series solution of the differential equation $(1-x^2)y^{//}-2xy^{/}+2y=0$, about the point $x=0$. **(10)**

(b) Solve $Z(x+y)\dfrac{\partial Z}{\partial x}+Z(x-y)\dfrac{\partial Z}{\partial y}=(x^2+y^2)$. **(10)**

Q.No.3. **(a)** Classify the following equations: **(5)**

(i) $\dfrac{\partial^2 Z}{\partial x^2}+x^2\dfrac{\partial^2 Z}{\partial y^2}-\dfrac{1}{x}\dfrac{\partial Z}{\partial x}=0$

(ii) $x^2\dfrac{\partial^2 Z}{\partial x^2}+2xy\dfrac{\partial^2 Z}{\partial x\partial y}+y^2\dfrac{\partial^2 Z}{\partial y^2}=4x^2$

(b) Solve: $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}, -1<x<1, t>0$ **(15)**

$u(-1,t)=u(1,t)$; $\dfrac{\partial u}{\partial x}(-1,t)=\dfrac{\partial u}{\partial x}(1,t)$ for $t>0$

$u(x, o)=x+1$, $-1<x<1$.

SECTION-B

Q.No.4. **(a)** Highlight the difference between a vector and a tensor. What happens if we permute the subscripts of a tensor? **(5)**

(b) Transform $g^{ab}=\begin{pmatrix}1 & 0\\ 0 & 1/r^2\end{pmatrix}$ into Cartesian coordinates. **(15)**

Q.No.5. **(a)** Workout the Christoffel symbols for the metric tensor $g_{ab} = \begin{pmatrix} a^2 & 0 \\ 0 & a^2 \sin^2\theta \end{pmatrix}$ **(10)**

(b) Workout the two dimensional metric tensor for the coordinates p and q given by $p = (xy)^{\frac{1}{3}}, \; q = \left(x^2 / y \right)^{1/3}$ **(10)**

SECTION-C

Q.No.6. **(a)** Solve the following system of equations by Jacobi iteration method: **(10)**

$10x + y - 2z = 7.74$
$x + 12y + 3z = 39.66$
$3x + 4y + 15z = 54.8$

(b) Solve $Sin x = 1 + x^3$ Using Newton-Raphson method. **(10)**

Q.No.7. **(a)** Find the root of $xe^x = 3$ by regular falsi method correct to three decimal places. **(10)**

(b) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ using **(5+5)** **(10)**

(i) Trapezoidal rule and
(ii) Simpson's rule.

Q.No.8. **(a)** Find the real root of the equation $Cos x = 3x - 1$ correct to seven decimal places by the iterative method. **(10)**

(b) Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the values of x and y are given below: **(10)**

X	5	6	9	11
Y	12	13	14	16

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2014

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper.**
(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B. ALL** questions carry **EQUAL** marks.
(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(iv) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** prove that $curl(¤\vec{F}) = (grad¤)$ x $\vec{F}$. If $\vec{F}$ is irrotational and ¤ (x, y, z) is a scalar function. **(10)**

(b) Determine whether the line integral: **(10)**
$\int_c (2xyz^2 dx + (x^2 z^2 + zCosyz)dy + (2x^2 yz + yCosyz)dz$ is independent of the path of integration? If so, then compute it from (1,0,1) to $(0, \frac{®}{2}, 1)$.

Q. No. 2. **(a)** State and prove Stoke's Theorem. **(10)**

(b) Verify Stoke's Theorem for the function $F = x^2 i - xy\, j$ integrated round the square in the plane z=0 and bounded by the lines x = y = 0, x = y = a. **(10)**

Q. No. 3. **(a)** Three forces act perpendicularly to the sides of a triangle at their middle points and are proportional to the sides. Prove that they are in equilibrium. **(10)**

(b) Three forces P, Q, R act along the sides BC, CA, AB respectively of a triangle ABC. Prove that, if P Sec A + Q Sec B + R Sec C = 0, then the line of action of the resultant passes through the orthocentre of the triangle. **(10)**

Q. No. 4. **(a)** Find the centroid of the surface formed by the revolution of the cardioide $r = a(1 + Cos¯)$ about the initial line. **(10)**

(b) A uniform ladder rests with its upper end against a smooth vertical wall and its foot on rough horizontal ground. Show that the force of friction at the ground is $\frac{1}{2} W \tan¯$, where W is the weight of the ladder and ¯ is its inclination with the vertical. **(10)**

Q. No. 5. **(a)** Define briefly laws of friction give atleast one example of each law. **(10)**

(b) A uniform semi-circular wire hangs on a rough peg, the line joining its extremities making an angle of 45^o with the horizontal. If it is just on the point of slipping, find the coefficient of friction between the wire and the peg. **(10)**

APPLIED MATHEMATICS, PAPER-I

SECTION-B

Q. No. 6. **(a)** If a point P moves with a velocity v given by $v^2=n^2(ax^2+2bx+c)$, show that P executes a simple harmonic motion. Find the center, the amplitude and the time-period of the motion. **(10)**

(b) A particle P moves in a plane in such a way that at any time t its distance from a fixed point O is $r= at+bt^2$ and the line connecting O and P makes an angle $\overline{\ } = ct^{\frac{3}{2}}$ with a fixed line OA. Find the radial and transverse components of the velocity and acceleration of the particle at $t=1$. **(10)**

Q. No. 7. **(a)** A particle of mass m moves under the influence of the force $F = a(i\,Sin\,\mu\,t + j\,Cos\,\mu\,t)$. If the particles is initially at rest on the origin, prove that the work done upto time t is given by $\frac{a^2}{m\mu^2}(1-Cos\,\mu\,t)$, and that the instantaneous power applied is $\frac{a^2}{m\mu^2}Sin\,\mu\,t$. **(10)**

(b) A battleship is streaming ahead with speed V, and a gun is mounted on the battleship so as to point straight backwards, and is set an angle of elevation a, if v_o is the speed of projection relative to the gun, show that the range is $\frac{2v_o}{g}Sin\sim(v_oCos\sim -V)$. Also prove that the angle of elevation for maximum range is $arcCos\left(\frac{V-\sqrt{V^2-8v_0^2}}{4v_0}\right)$. **(10)**

Q. No. 8. **(a)** Show that the law of force towards the pole, of a particle describing the curve $r^n = a^nCos\,n\overline{\ }$ is given by $f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$. **(10)**

(b) A bar 2 ft. long of mass 10 Ib., lies on a smooth horizontal table. It is struck horizontally at a distance of 6 inches from one end, the blow being perpendicular to the bar. The magnitude of the blow is such that it would impart a velocity of 3 ft./sec. to a mass of 2 Ib. Find the velocities of the ends of the bar just after it is struck. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2014

Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q.No**. in the **Answer Book** in accordance with **Q.No**. in the **Q.Paper.**
(ii) Attempt FIVE questions in all by selecting **TWO** questions from **SECTION-A, ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C. ALL** questions carry **EQUAL** marks.
(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(iv) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. (a) Solve the initial-value problem **(10)**

$$\frac{dy}{dx} = \frac{1}{x + y^2}; \quad y(-2) = 0 .$$

(b) Initially there were 100 milligrams of a radioactive substance present. After 6 hours the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours. **(10)**

Q. No. 2. (a) Solve $(x^2 + 1)y'' + xy' - y = 0$. **(10)**

(b) Obtain the partial differential equation by elimination of arbitrary functions, **(10)**
$a \sin x + b \cos y = z$ (take z as dependent variable).

Q. No. 3. (a) Solve the partial differential equation $u_{xx} + u_{yy} = u_t$, **(10)**

subject to the conditions
$$u(0, y, t) = u(a, y, t) = 0$$
$$u(x, 0, t) = u(x, a, t) = 0$$
and the initial condition, $u(x, y, 0) = ¤(x, y)$.

(b) Solve $r + (a + b)s + abt = xy$ by Monge's method. **(10)**

SECTION-B

Q. No. 4. (a) Prove that if $A_i, B_j, and\ C_k$ are three first order tensors, then their product $A_i B_j C_k \ (i, j, k = 1, 2, 3)$ is a tensor of order 3, while $A_i B_j C_k (i, j = 1, 2, 3)$ form a first order tensor. **(10)**

(b) If $A_{i_1 i_2 i_3 \cdots i_n}$ is a tensor of order *n*, then its partial derivative with respect to x_p that is $\frac{\partial}{\partial x_p} A_{i_1 i_2 i_3 \cdots i_n}$ is also a tensor of order *n+1*. **(10)**

Q. No. 5. **(a)** Show that the transformation $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -3 & -6 & -2 \\ -2 & 3 & -6 \\ 6 & -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is orthogonal and right-handed. **(10)**

A second order tensor A_{ij} is defined in the system $Ox_1x_2x_3$ by
$A_{ij} = x_i x_j \quad i, j = 1, 2, 3.$ Evaluate its components at the point P where
$x_1 = 0,\ x_2 = x_3 = 1.$ Also evaluate the component A'_{11} of the tensor at P.

(b) The Christofell symbols of the second kind denoted by $\begin{Bmatrix} m \\ ij \end{Bmatrix}$ are defined **(10)**

$\begin{Bmatrix} m \\ ij \end{Bmatrix} = g^{mk}\left[ij, k\right] \quad (i, j, k = 1, 2, \ldots n).$

Prove that (i) $\begin{Bmatrix} m \\ ij \end{Bmatrix} = \begin{Bmatrix} m \\ ji \end{Bmatrix}$, (ii) $[ij, k] = g_{mk}\begin{Bmatrix} m \\ ij \end{Bmatrix}$,

(iii) $\dfrac{\partial g^{ij}}{\partial x^k} = -g^{im}\begin{Bmatrix} i \\ km \end{Bmatrix} - g^{jm}\begin{Bmatrix} i \\ km \end{Bmatrix}.$

SECTION-C

Q. No. 6. **(a)** Apply Newton-Raphson's method to determine a root of the equation $f(x) = \cos x - xe^x = 0$ such that $\left|f(x^*)\right| < 10^{-8}$,where x^* is the approximation to the root. **(10)**

(b) Consider the system of the equations **(10)**

$2x_1 - x_2 + 0x_3 = 7$

$-x_1 + 2x_2 - x_3 = 1,$

$0x_1 - x_2 + 2x_3 = 1$

Solve the system by using Gauss-Seidel iterative method and perform three iterations.

Q. No. 7. **(a)** Use the trapezoidal and Simpson's rules to estimate the integral **(10)**

$\int_1^3 f(x)dx = \int_1^3 (x^3 - 2x^2 + 7x - 5)dx$.

(b) Find the approximate root of the equation $f(x) = 2x^3 + x - 2 = 0$. **(10)**

Q. No. 8. **(a)** Find a 5th degree polynomial which passes through the 6 points given below. **(10)**

x	1.0	2.0	4.0	5.0	7.0	8.0
$f(x)$	-9	-41	-189	-173	9	523

(b) Determine the optimal solution graphically to the linear programming problem, **(10)**

$$\begin{aligned} &\textit{Minimize} \quad z = 3x_1 + 6x_2 \\ &\textit{subject to} \quad 4x_1 + x_2 \ge 20 \\ &\qquad x_1 + x_2 \le 20 \\ &\qquad x_1 + x_2 \ge 10 \\ &\qquad x_1, x_2 \ge 0 \end{aligned}$$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2015

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-I

Q. No. 1 **(a)** Prove that $(\vec{A}+\vec{B})\cdot(\vec{B}+\vec{C})\times(\vec{C}+\vec{A}) = 2[\vec{A}\cdot(\vec{B}\times\vec{C})]$. **(10)**

(b) If $\vec{A} = (x-3y)\hat{i} + (y-2x)\hat{j}$, evaluate $\oint_c \vec{A}.d\vec{r}$ where c is an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in the xy- plane traversed in the positive direction. **(10)**

Q. No. 2 **(a)** Determine the expression for divergence in orthogonal curvilinear coordinates. **(10)**

(b) Determine the unit vectors in spherical coordinate system. **(10)**

Q. No. 3 **(a)** A particle moves from rest at a distance "*a*" from a fixed point *O* where the acceleration at distance *x* is $« x^{-\frac{5}{3}}$. Show that the time taken to arrive at *O* is given by an equation of the form $t = A\frac{a^{\frac{4}{3}}}{\sqrt{«}}$, where *A* is a number. **(10)**

(b) Three forces *P*, *Q*, *R* acting at a point, are in equilibrium, and the angle between *P* and *Q* is double of the angle between *P* and *R*. Prove that $R^2 = Q(Q-P)$. **(10)**

Q. No. 4 **(a)** *AB* and *AC* are similar uniform rods, of length *a*, smoothly joined at *A*.*BD* is a weightless bar, of length *b*, smoothly joined at *B*, and fastened at *D* to a smooth ring sliding on *AC*. The system is hung on a small smooth pin at *A*. Show that the rod *AC* makes with the vertical an angle $\tan^{-1}\frac{b}{a+\sqrt{a^2-b^2}}$. **(10)**

(b) Find the centroid of the arc of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant. **(10)**

Q. No. 5 **(a)** A hemispherical shell rests on a rough inclined plane whose angle of friction is a . Show that the inclination of the plane base to the horizontal cannot be greater than $\sin^{-1}(2\sin a)$. **(10)**

(b) A regular octahedron formed of twelve equal rods, each of weight *w*, freely jointed together is suspended from one corner. Show that the thrust in each horizontal rod is $\frac{3}{2}\sqrt{2}w$. **(10)**

APPLIED MATHEMATICS, PAPER-I

SECTION-II

Q. No. 6 **(a)** A particle is moving with uniform speed v along the curve $x^2 y = a(x^2 + \frac{a^2}{\sqrt{5}})$. **(10)**

Show that its acceleration has the maximum value $\frac{10v^2}{9a}$.

(b) Discuss the motion of a particle moving in a straight line if it starts from rest at a distance a from a point O and moves with an acceleration equal to « times its distance from O. **(10)**

Q. No. 7 **(a)** Prove that the force field **(10)**

$F = (y^2 - 2xyz^3)i + (3 + 2xy - x^2 y^3)j + (6z^3 - 3x^2 yz^2)k$

is conservative, and determine its potential.

(b) The components of velocity along and perpendicular to the radius vector form a fixed origin are respectively ª r^2 and «¯^{2}. **(10)**

Find the polar equation of the path of the particle in terms of r and ¯.

Q. No. 8 **(a)** A particle is projected horizontally from the lowest point of a rough sphere of radius a. After describing an arc less than a quadrant, it returns and comes to rest at the lowest point. Show that the initial speed must be $(\sin \sim)\sqrt{\frac{2ag(1+«^2)}{(1-2«^2)}}$, **(10)**

Where « is the coefficient of friction and a~ is the arc through which the particle moves.

(b) The law of force is Mu^2 and a particle is projected from or apse at distance a. Find the orbit when the velocity of the projection is $\frac{\sqrt{M}}{a^2}$. **(10)**

FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
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UNDER THE FEDERAL GOVERNMENT, 2015

Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A, ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) Candidate must write **Q.No**. in the **Answer Book** in accordance with **Q.No**. in the **Q.Paper.**
(iii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iv) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(v) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(vii) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** Solve the initial value problem. **(10)**

$$\frac{dy}{dx}+\frac{y}{2x}=\frac{x}{y^3}, y(1)=2$$

(b) Solve $y''-4y'+4y=e^{2x}$ **(10)**

Q. No. 2. Solve the following equations: **(10)**

(a) $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$

(b) $\frac{d^3y}{dx^3}+\frac{dy}{dx}=\cos ecx$ **(10)**

Q. No. 3. **(a)** Classify the following: **(5 each)** **(10)**

(i) $x^2U_{xx}+(a^2-y^2)U_{yy}=0 \quad , \quad -\infty<x<\infty,\ -a<y<a$

(ii) $U_{xx}-6U_{xy}+9U_{yy}+3y=0$

(b) Solve **(10)**

$$\frac{\partial^2 u}{\partial^2 t}=\frac{\partial^2 u}{\partial^2 x} \qquad 0<x<5$$
$$u(0,t)=u(5,t)=0$$
$$u(x,0)=x^2(x-5)$$
$$u_t(x,0)=0$$

SECTION-B

Q. No. 4. **(a)** Prove that if A_i and B_j are two first order tensors, then their product $A_iB_j\,(i,j=1,2,3)$ is a second order tensor. **(7)**

(b) If ¤(x_1,x_2,x_3) is a scalar point function then $\frac{\partial ¤}{\partial x_i}$ are the components of a first order tensor. **(7)**

(c) Find the invariant of the following second order tensor **(6)**

$$\begin{bmatrix} 2 & 4 & -1 \\ 6 & -7 & 10 \\ 3 & -4 & 6 \end{bmatrix}$$

APPLIED MATHEMATICS, PAPER-II

Q. No. 5. **(a)** Verify that the transformation **(7)**

$$x_1' = \frac{1}{15}(5x_1 - 14x_2 + 2x_3)$$
$$x_2' = -\frac{1}{3}(2x_1 + x_2 + 2x_3)$$
$$x_3' = \frac{1}{15}(10x_1 + 2x_2 - 11x_3)$$

Is orthogonal and right handed. A vector field $\vec{A}$ is defined in the system

$Ox_1x_2x_3$ by $A_1 = x_1^2, A_2 = x_2^2, A_3 = x_3^2$

Evaluate the components A_j' of the vector field in the new system $Ox_1'x_2'x_3'$.

(b) Prove that any second order tensor A_{ij} can be written as the sum of a deviator and an isotropic tensor. **(7)**

(c) If $a_{ij} = a_{ji}$ are constants. Calculate. **(6)**

$$\frac{\partial^2}{\partial X_k \partial X_m}(a_{ij} X_i X_j)$$

SECTION-C

Q. No. 6. **(a)** Find the real root of the equation by using Newton – Raphson's method. **(10)**

$3x - \cos x - 1 = 0$

(b) Solve the following system of equations by Gauss-Seidel method.
Take initial approximation as $x_1 = 0,\ x_2 = 0,\ x_3 = 0$. Perform 3 Iterations. **(10)**

$20x_1 + x_2 - 2x_3 = 17$

$3x_1 + 20x_2 - x_3 = -18$

$2x_1 - 3x_2 + 20x_3 = 25$

Q. No. 7. **(a)** Find the real root of the equation $x^3 - 4x - 9 = 0$ by Regular falsi method. Take the interval of the root as (2,3) and perform 4 iterations. . **(10)**

(b) Find a polynomial which possess through the following points: **(10)**

x:	-1	0	1	2
$f(x)$:	2	1	2	5

Q. No. 8. **(a)** Use the langrage's Interpolation formula to find the value *f*(12) if the values of *x* and *f(x)* are given below **(10)**

x:	5	7	11	13
f(x)	150	392	1452	2366

(10)

(b) Evaluate $\int_0^3 x\sqrt{1+x^2}\,dx$ using $\frac{1}{3}$ Simpson's rule and trapezoidal rule for $n = 6$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2016
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed**.

Q. No. 1. **(a)** Prove that $\nabla.\left[\frac{f(r)\,\vec{r}}{r}\right] = \frac{2}{r} f(r) + f'(r)$ **(10)**

(b) Verify Stokes' theorem for $\vec{A} = (2x - y)\,\hat{i} - y z^2\,\hat{j} - y^2 z\,\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary . **(10)**

Q. No. 2. **(a)** Forces P, Q, R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant force is **(10)**

$$\sqrt{P^2 + Q^2 + R^2 - 2\,Q\,R\cos A - 2\,R\,P\cos B - 2\,P\,Q\cos C}$$

(b) Find the distance from the cusp of the centroid of the region bounded by the cardioid $r = a(1 + \cos\theta)$. **(10)**

Q. No. 3. **(a)** A particle describes simple harmonic motion in such a way that its velocity and acceleration at a point P are u and f respectively and the corresponding quantities at another point Q are v and g. Find the distance PQ. **(10)**

(b) Derive the radial and transverse components of velocity and acceleration of a particle. **(10)**

Q. No. 4. Solve the following differential equations:

(a) $\frac{dy}{dx} + \frac{y}{x} = x^3 y^4$ **(10)**

(b) $(D^2 - 5D + 6)\,y = x^3 e^{2x}$ **(10)**

Q. No. 5. **(a)** Solve the differential equation using the method of variation of parameters **(10)**

$$\frac{d^2 y}{dx^2} + y = \tan x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(b) Solve the Euler – Cauchy differential equation $x^2 y'' - 3xy' + 4y = x^2 \ln x$. **(10)**

Q. No. 6. **(a)** Find the Fourier series of the following function: **(10)**

$$f(x) = \begin{cases} -x & \text{if } -\pi < x < 0 \\ x & \text{if } 0 < x < \pi \end{cases}$$

(b) Solve the initial - boundary value problem: **(10)**

Q. No. 7. **(a)** Apply Newton – Raphson method to find the smaller positive root of the equation $x^2 - 4x + 2 = 0$ **(10)**

(b) Solve the following system of equations by Gauss – Seidel iterative method by taking the initial approximation as $x_1 = 0, \ x_2 = 0, \ x_3 = 0$: **(10)**

$$5x_1 + x_2 - x_3 = 4$$
$$x_1 + 4x_2 + 2x_3 = 15$$
$$x_1 - 2x_2 + 5x_3 = 12$$

Q. No. 8.

(a) Approximate $\int_0^1 \frac{dx}{1 + x^2}$ using **(10)**

(i) Trapezoidal rule with $n = 4$ (ii) Simpson's rule with $n = 4$

Also compare the results with the exact value of the integral.

(b) Apply the improved Euler method to solve the initial – value problem: **(10)**

$y' = x + y, \quad y(0) = 0$

by choosing $h = 0.2$ and computing $y_1, \ldots, y_5$.

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
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(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed**.

Q. No. 1. **(a)** Suppose $\mathbf{r}(t) = (e^t \cos t)\hat{i} + (e^t \sin t)\,\hat{\mathbf{j}}$. Show that the angle between $\mathbf{r}$ and $\dfrac{d^2\mathbf{r}}{dt^2}$ never changes. What is the angle? **(10)**

(b) Using divergence theorem of Gauss, Evaluate $\iint_S x^2 dxdy + x^2 ydzdx + x^2 zdxdy$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$, $(0 \le z \le b)$ and the circular disks $z = 0$ and $z = b$, $(x^2 + y^2 \le a^2)$. **(10)**

Q. No. 2. **(a)** A 100 Kg wooden crate rests on a wooden ramp with an adjustable angle of inclination. Draw a free body diagram of the crate. If the angle of the ramp is set to 10^0, find the perpendicular and parallel components of the crate's weight to the ramp. Also find the static friction force between the crate and the ramp. At what angle will the crate just begin to slip? (coefficient of static friction μ_s between wood block and wood surface is 0.28) **(10)**

(b) A ladder having a uniform density and a mass m rests against a frictionless vertical wall at an angle of 60^0. The lower end rests on a flat surface where the coefficient of static friction is 0.40. A person of mass M=2m attempts to climb the ladder. What fraction of the length L of the ladder will the person have reached when the ladder begins to slip? **(10)**

Q. No. 3. **(a)** A particle moving in a straight line starts with a velocity u and has acceleration v^3, where v is the velocity of the particle at time t. Find the velocity and the time as functions of the distance travelled by the particle. **(10)**

(b) A particle describing simple harmonic motion has velocities 5 ft/sec and 4 ft/sec when its distance from the centre are 12 ft and 13 ft respectively. Find the time-period of motion. **(10)**

Q. No. 4. **(a)** Check for the exactness and solve the following ordinary differential equation. **(10)**
$(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

(b) Solve the following second order differential equation: **(10)**
$$\frac{d^2y}{dx^2} + 4x = \sec 2x$$

Q. No. 5. **(a)** Solve the initial value problem. **(10)**
$$\frac{d^2y}{dx^2} = 2-6x,\ y'(0) = 4,\ y(0) = 1$$

(b) Solve the following Boundary Value Problem. **(10)**
$y' + 4y = 0$, $y(0) = -2$, $y(2\pi) = -2$
How many solutions do you get for this problem?

APPLIED MATHEMATICS

Q. No. 6. **(a)** Compute the Fourier series for the function x^2 on the interval $0<x<L$, using as a basis of function with boundary conditions $u'(0)=0$ and $u'(L) = 0$. Sketch the partial sums of the series for 1, 2, 3 terms. **(10)**

(b) Find a solution to the following partial differential equation that will also satisfy the boundary conditions. **(10)**

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}\ , \quad u(x,0) = f(x)\ ,\ u(0,t) = 0,\ u(L,t) = 0$$

Q. No. 7. **(a)** Use bisection and false position methods to locate the root of $f(x) = x^{10} - 1$, between 0 and 1.3. Which of the two methods is better and why? **(10)**

(b) Use Lagrange interpolation polynomial of the first and second order to evaluate f(x) at x=15 for the following data; **(10)**

x:	0	20	40
f(x):	3.850	0.800	0.212

Q. No. 8. **(a)** Use Trapezoidal Rule with 3 segments to evaluate. **(10)**

$$\int_0^{0.8} x\cdot e^{2\cdot x}\,dx$$

Also find the true solution and the percentage error.

(b) Consider the function $f(x) = xe^x$. Obtain approximations to $f'(2)$ with $h = 0.5$ using forward, backward and central difference formulas. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2018
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed**.

Q. No. 1. **(a)** If $\psi = Sin\frac{(kr)}{r}$, then show that $\nabla^2\psi + k^2\psi = 0$. **(10)**

(b) Calculate the Line Integral $\int_C A.dr$, $where\ A = \frac{-yi + xj}{x^2 + y^2}$, and the curve C is given by the equations $x^2 + y^2 = a^2 \quad and \quad Z = 0$. **(10)**

Q. No. 2. **(a)** Forces of magnitude P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD, of sides **a**, and forces each of magnitude $(8\sqrt{2}\ )$ P act along the diagonals BD, AC. Find the magnitude of the resultant force and distance of its line of action from A. **(10)**

(b) A uniform ladder, of length 70 feet, rests against a vertical wall with which it makes an angle of 45^0, the coefficient of friction between the ladder and the wall and the ground respectively being $\frac{1}{3}$ and $\frac{1}{2}$. If a man, whose weight is one half that of the ladder, ascends the ladder, where will he be when the ladder slips? **(10)**

Q. No. 3. **(a)** A particle moves in a straight line with an acceleration kv^3. If its initial velocity is u, find the velocity and the time spent when the particle has travelled a distance x. **(10)**

(b) Derive the Tangential and Normal components of the velocity and acceleration. **(10)**

Q. No. 4. **(a)** Solve the following Cauchy- Euler Equation **(10)**

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0.$$

(b) Convert the following Bernoulli Differential Equation into standard form and then solve. **(10)**

$$\frac{dy}{dx} + \left(\frac{xy}{1-x^2}\right) = xy^{\frac{1}{2}}.$$

Q. No. 5. **(a)** Convert the following Ordinary Differential Equation into standard form and then solve using Method of Variation of Parameters. **(10)**

$$x^2y'' - 3xy' + 3y = 2x^4e^x$$

(b) Check whether the following Ordinary Differential Equation is an Exact Equation or not. If yes, then solve. **(10)**

$$(3x^2y + 2)\,dx + (x^3 + y)\,dy = 0$$

Q. No. 6. **(a)** Find the Fourier Series of f on the given interval. **(10)**

$$f(x)=\begin{cases}-1, & -\pi<x<0 \\ 1, & 0\le x<\pi\end{cases}$$

(b) Solve the following Partial Differential Equation subject to the conditions given. **(10)**

$$a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}, \qquad 0<x<L, \qquad t>0,$$

$$u(0,t)=0, \qquad u(L,t)=0, \qquad t>0,$$

$$u(x,0)=f(x),\ \frac{\partial u}{\partial t}=g(x)[at\ time\ t=0], \qquad and \quad 0<x<L.$$

Q. No. 7. **(a)** Use Newton-Raphson method to find solution accurate to within 10^{-4} for the non-linear equation. **(10)**

$$x^3-2x^2-5=0, \qquad I=[1,4]$$

(b) Use Lagrange Interpolating polynomial of degree two to approximate $f(8.4)$, If **(10)**

$$f(8.1)=16.94410, \quad f(8.3)=17.56492, \quad f(8.6)=18.50515, \quad f(8.7)=18.82091.$$

Q. No. 8. **(a)** Approximate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule and Simpson's rule with $n=4$. **(10)**

Also compare your results with the exact value of the integral.

(b) Use Euler's method to approximate the solution of the following initial value problem. **(10)**

$$y'=\frac{1+y}{t}, \qquad 1\le t\le 2, \qquad with\ \ y(1)=2, \qquad h=0.25$$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2019
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
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(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) Use of Calculator is allowed.

Q. No. 1. **(a)** Find the directional derivative of $f(x,y,z) = xy^2 + yz^2$ at the point (2,-1, 1) in the direction of the vector $i + 2j + 2k$? **(10)**

(b) Evaluate $\int_c (xy + y^2)dx + x^2dy$ where c is bounded by the line $y = x$ and the curve $y = x^2$ **(10)**

Q. No. 2. **(a)** Find the constants a, b, and c so that **(10)**

$$F=(x+2y+az)\,i + (bx - 3y - z)\,j + (4x + cy + 2z)\,k$$

is irrotational and hence find the function ψ such that $F = \nabla \psi$

(b) The forces F_1, F_2, F_3, F_4, F_5 and F_6 act along the sides of a regular hexagone taken in order. Verify that all the forces will be in equilibrium if, **(10)**

$$\sum F = 0, \text{ and } F_1 - F_4 = F_3 - F_6 = F_5 - F_2 .$$

Q. No. 3. **(a)** A system of forces acts on a plate in the form of an equilateral triangle of side 2a. The moment of the forces about the three vertices are M_1, M_2 and M_3 respectively. Find the magnitudes of the resultant. **(10)**

(b) If a particle P move with a velocity V given by $V^2 = n^2(ax^2 + 2bx + c)$. Show that P executes a simple harmonic motion. Find the centre, the amplitude and the time period of the motion? **(10)**

Q. No. 4. **(a)** What is the difference between linear differential equation and Bernoulli's equation? Also find the solution of the following differential equation. **(10)**

$$x\left[\frac{dy}{dx} + y\right] = 1 - y$$

(b) Use the method of undetermined coefficient to solve the following differential equation. **(10)**

$$y'' - 3y' + 2y = 2x^3 - 9x^2 + 6x$$

Q. No. 5. **(a)** Solve the equation **(10)**

$$0 = \frac{1}{2} + \frac{1}{4}x^2 - x\sin x - \frac{1}{2}\cos 2x \qquad with\ x_0 = \frac{\pi}{2}$$

(b) Derive two point Gaussian integration formula for the following integral and use it to solve the integral. **(10)**

$$\int_1^{1.6} \frac{2x}{x^2 - 4}\,dx$$

Q. No. 6. **(a)** Determine the second degree polynomials by using Newton's method. Also estimate the value of $f(0.1)$ and $f(0.5)$ for the data. **(10)**

x	0.0	0.2	0.4	0.6
$f(x)$	15.0	21.0	30.0	51.0

(b) Does the dominate diagonal is necessary for finding the numerical solution of system of linear equations by using Gauss Jacobi's and Gauss Seidal methods. Explain the reason. In what conditions a numerical method is used instead of analytical method? Find the solution of the following system by performing three itrations of Gauss Seidal method. **(10)**

$$6x - 3y + z = 11$$
$$2x + y - 8z = 15$$
$$x - 7y + z = 10$$

Q. No. 7. **(a)** Define even function and odd function with examples. Verify that the Fourier **(10)**

Series for the function $f(x) = \begin{cases} 0 & \text{When } 0 < x < \pi \\ 1 & \text{When } \pi < x < 2\pi^3 \end{cases}$

is $f(x) = \frac{1}{2} - \frac{2}{\pi}(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x...............)$

(b) Solve the following partial differential equation by using method of separable variable. **(10)**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\,, \qquad given \qquad u(x,o) = 6e^{-3x}$$

Q. No. 8. **(a)** The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4, and the Simpson's rule gives value 2, what is the value of $f(1)$? **(10)**

(b) Find the first two derivatives at x=1.1 and x =1 from the following data table. **(10)**

x	1	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.000	0.1280	0.5440	1.2960	2.4320	4.000

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2020
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed**.

Q. No. 1. **(a)** Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ **(10)**

(b) Evaluate $\iint_S \underline{A}.\bar{n}\, ds$ where $\overline{A}=18z\underline{i}-12\underline{j}+3y\underline{k}$ and S is that part of the plane $2x+3y+6z=12$ which is located in the 1st octant. **(10)**

Q. No. 2. A particle P of mass m slides down a frictionless inclined plane AB of an angle α with the horizontal. If it starts from rest at the top A, find (a) the acceleration (b) the velocity and (c) the distance travelled after time t. **(20)**

Q. No. 3. **(a)** Discuss the motion of a particle moving in a straight line if it starts from rest at a distance 'a' from a point O and moves with an acceleration equal to k times its distance from O. **(10)**

(b) Find radial and transversal components of velocity and acceleration. **(10)**

Q. No. 4. **(a)** Solve $\dfrac{d^2y}{dx^2}+y=Co\sec x$ **(10)**

(b) Solve $dy+\dfrac{y-Sinx}{x}dx=0$ **(10)**

Q. No. 5. **(a)** Solve the initial value problem **(10)**

$$x(2+x)\frac{dy}{dx}+2(1+x)y=1+3x^2, \qquad y(-1)=1$$

(b) Find the general solution of the equation **(10)**

$$(D^3-2D+1)y=2x^3-3x^2+4x+5$$

Q. No. 6. **(a)** Find the Fourier series of f: **(10)**

$$f(x)=\begin{cases} x, 0<x<1 \\ 0, 1<x<2 \end{cases}$$

(b) Solve the boundary value problem $\dfrac{\partial^2 u}{\partial x^2}=\dfrac{1}{k}\dfrac{\partial u}{\partial t}$ **(10)**

Satisfying u(o,t)=u(l,t)=0 and u(x,o)=lx-x^2

Q. No. 7. **(a)** By using regular Falsi method, solve **(10)**
$Logx - Cosx = 0$

(b) Find the value of $f(7.5)$ by using Newton Gregory Backward Difference Interpolation formula. **(10)**
X: 5, 6.1, 6.9, 8, 8.6
$f(x): 3.49, 4.82, 5.96, 7.5, 8.2$

Q. No. 8. **(a)** Applying the Taylor series method, compute **(10)**

$$\int_{o}^{x} \frac{Sin\,t}{t} dt \quad for \;\; x = 0\ (0.1)1$$

(b) Use fourth order RK method to solve **(10)**

$$\frac{dy}{dx} = t + y \;\; ; \;\; y(0) = 1 \quad from\ t = 0 \;\; to\ t = 0.4\ taking\ h = 0.4$$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2022 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:
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- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed**.

Q. No. 1. **(a)** Let u=[y, z, x] and v=[yz, zx, xy], f = xyz and g = x+ y+ z. Find div (grad (*fg*)). **(10)**

(b) Evaluate $\int_C F(r).dr$ counter clockwise around the boundary *C* of the region *R* by Green's theorem, where **(10)**

$$F = [y, -x], C\ the\ circle\ x^2 + y^2 = 1/4$$

Q. No. 2. **(a)** Three forces P, Q, R, acting at a point, are in equilibrium, and the angle between P and Q is double of the angle between P and R. Prove that $R^2 = Q(Q - P)$. **(10)**

(b) Find the centre of mass of a semi-circular lamina of radius a whose density varies as the square of the distance from the centre. **(10)**

Q. No. 3. **(a)** A particle moves in such a way that its position vector at time t is **(10)**

$$r = (a\cos nt)\mathrm{i} + (b\sin nt)\mathrm{j},$$

Where a, b, n are constants and a>b>0. Show that the path of the particle is an ellipse of semi-major and minor axes a, b respectively, and that the field of force is directed towards the centre of the ellipse. Also find the maximum speed.

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a, whose centre is at a height h vertically above a point O of the ground . If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O, show that Y satisfies the equation **(10)**

$$KY^2 + Y(a^2 - 2hK) + K(h^2 - a^2) = 0,$$

Where $K = h + \frac{ga^2}{2{v_0}^2}$.

Q. No.4. **(a)** Solve the given initial-value problem. Give the largest interval *I* over which the solution is defined. **(10)**

$$xy' + y = e^x,\ y(1) = 2.$$

(b) Find the general solution of the given higher-order differential equation. **(10)**

$$y''' - 4y'' - 5y' = 0$$

Q. No. 5. **(a)** Find two power series solutions of the given differential equation about the ordinary point x=0. **(10)**

$$y'' - 2xy' + y = 0.$$

(b) Find the general solution of the given Bessel's equation on (0, ∞). **(10)**

$$x^2y'' + xy' + (9x^2 - 4)y = 0$$

Q. No. 6. **(a)** Find the Fourier series of the given function f(x), which is assumed to have the period 2π. Show the details of your work. **(10)**

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

(b) Find u(x,t) for the string of length L=1 and $c^2=1$ when the initial velocity is zero and the initial deflection with small k (say, 0.01) is $kx(1-x)$. **(10)**

Q. No. 7. **(a)** Use the Bisection method to determine an approximation to the root of the given function in the interval [1,2] that is accurate to at least within 10^{-4}. **(10)**

$$f(x) = x^3 + 4x^2 - 10 = 0.$$

(b) Values for $f(x) = xe^x$ are given in the following table. Use all the applicable three-point and five-point formulas to approximate $f'(2.0)$. **(10)**

x	1.8	1.9	2.0	2.1	2.2
f(x)	10.889365	12.703199	14.778112	17.148957	19.85503

Q. No. 8. **(a)** Use the Modified Euler method to approximate the solution to each of the following initial-value problem, **(10)**

$$y' = -5y + 5t^2 + 2t, \quad 0 \le t \le 1, \qquad y(0) = \frac{1}{3}, with\ h = 0.1$$

(b) Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1, 2]. Use $p_0 = 1$. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:
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- **(iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed**.

Q. No. 1. **(a)** Forces of magnitudes P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD, of side a and forces each of magnitude $(8\sqrt{2})$ P act along the diagonals BD, AC. Find the magnitude of the resultant force and the distance of its line of action from A. **(10)**

(b) A uniform rod AB of length a and weight W is freely hinged to a vertical wall at *A* and is maintained in equilibrium by a light string of length a fastened to *B* and to a point *C* at a distance *b* vertically above *A*. Prove that the reaction at the hinge *A* is **(10)**

$$W\frac{\sqrt{(a^2+2b^2)}}{2b}$$

and find the tension in the string.

Q. No. 2. **(a)** Use Runge-Kutta method of order two to solve the following differential equation at *x*=1.2 by taking *h*=0.1 **(10)**

$$\frac{dy}{dx}=\frac{3x+y}{x+2y} \quad y(1)=1\,.$$

(b) Find the first and second derivatives of *f*(*x*) at *x* = 3 from the following data using Newton's forward difference interpolation formula **(10)**

x	3	3.5	4	4.5	5	5.5
f(*x*)	4.1023	5.1047	8.1971	9.1096	4.1122	6.1148

Q. No. 3. **(a)** Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$. **(8)**

(b) Show that **(6)**

$$\nabla r^n = nr^{n-2}r$$

(c) Find the total work done in a moving particle in a force field given by $F=3xy\,\boldsymbol{i}-5\,z\,\boldsymbol{j}+10\,x\,\boldsymbol{k}$ along the curve $x=t^2+1, y=2t^2, z=t^3$ from $t=1$ to $t=2$. **(6)**

Q. No.4. **(a)** A particle P moves in a plane in such a way that at any time t, its distance from a fixed point *O* is $r=a\,t+b\,t^2$ and the line connecting *O* and *P* makes an angle $\theta=ct^{\frac{3}{2}}$ with a fixed line *OA*. Find the radial and transverse components of the velocity and acceleration of the particle at $t=1$. **(10)**

(b) Solve the following Bernouli's equation **(10)**

$$x\frac{dy}{dx}+y=\frac{1}{y^2}$$

Q. No. 5. **(a)** Solve the following differential equation **(10)**

$$x\,dy=(x\,sinx-y)\,dx$$

(b) Find the general solution of the higher order differential equation **(10)**

$$y'''+8y''=-6\,x^2+9x+2$$

Q. No. 6. **(a)** Find solution of $4y'' + y = 0$ in the form of power series in x. **(10)**

(b) Solve the following differential equation by variation of parameters **(10)**

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Q. No. 7. **(a)** Find real root of the equation $2x - 3\sin(x) - 5 = 0$ up to 4 decimal places by secant method. **(10)**

(b) Solve the following system of equations by Guass Seidel method. Perform only five iterations. **(10)**

$$\begin{aligned} 8x_1 - x_2 - x_3 &= 6 \\ x_1 + 6x_2 + x_3 &= 8 \\ x_1 - x_2 + 5x_3 &= 5 \end{aligned}$$

Q. No. 8. **(a)** Expand $f(x) = \sin x,\ 0 < x < \pi$, in a Fourier cosine series. **(10)**

(b) Use the method of separation of variables to find the solution of the following boundary value problem **(10)**

$$\nabla^2 u = \frac{\partial^2 u}{\partial\, x^2} + \frac{\partial^2 u}{\partial\, y^2} = 0, \qquad 0 \le x \le a, \qquad 0 \le y \le b$$

$$u_x(0,y) = 0, \quad u_x(a,y) = 0,$$
$$u_y(x,b) = 0, \quad u(x,0) = f(x).$$

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