FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2015

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-I

Q.No.1. (a) Let H be a subgroup of a group G. Prove that the normalizer of H in G (i.e. $N_G(H)$) is a subgroup of G. — 10

(b) Prove that a group of prime order is cyclic. — 10

Q.No.2. (a) Write three non-isomorphic groups of order 12. — 10

(b) Prove that a group G is isomorphic to a subgroup of group of automorphisms of G. — 10

Q.No.3. (a) Construct Cayley's table for Multiplication Modulo 7 of — 8 (4+3+1)

$$Z_7 - \{\bar{0}\} = \{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{4}, \bar{5}, \bar{6}\}.$$

Show that Z_7 is an integral domain. (You may use Cayley's table.)

Is Z_7 a field? Justify your answer.

(b) Give an example of zero divisor in $Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{4}, \bar{5}\}$. — 5 (2+1+2)

Is Z_6 an integral domain? Justify your answer.

(c) What is FIELD EXTENSION? — 7

Verify that the field $\mathbb{Q}[\sqrt{5}] = \{x + y\sqrt{5} : x, y \ \mathbb{Q}\}$ is an extension of $\mathbb{Q}$.

Q.No.4. (a) Show that $W = \left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{R}\right\}$ is a subspace of the vector space $\mathcal{M}_2(\mathbb{R})$ consisting of all 2×2 matrices over $\mathbb{R}$. — 10

(b) Prove that if a subset $\{v_1, v_2, \cdots, v_k\}$ of a vector space V is linearly dependent then one vector among $v_1, v_2, \cdots, v_k$ is linear combination of the remaining vectors. — 4

(c) — 6 (1+1+2+2)

(1) What is dimension of $\mathbb{R}^3$.

(2) Write a basis of $\mathbb{R}^3$.

(3) Is $\{(1,1,0), (1,1,2), (1,0,1), (0,1,2)\} \subseteq \mathbb{R}^3$ linearly dependent or independent? Justify your answer.

(4) Is $\{(0,0,0), (1,1,2), (1,0,1)\} \subseteq \mathbb{R}^3$ linearly dependent or independent? Justify your answer.

Q.No.5. (a) Define eigen value of a square matrix. 10
Find eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) Find reduced echelon form of the matrix 10

$$A = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 1 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

(c) Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be eigen values of a square matrix $A = [a_{ij}]$. What are $|A|$ and $trace(A)$ in terms of λ_i's?

SECTION-II

Q.No.6. (a) Find equations of tangent plane and normal line at a point (x_1, y_1, z_1) of ellipsoid 10

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

(b) Find equation of the ellipse centered at the origin, a focus at (3, 0) and vertex at (3, 0). 5

(c) Find the polar equation of a parabola $x = 8y^2$. 5

Q.No.7. (a) Find the equation of elliptic paraboloid

$$x = y^2 + z^2$$

In spherical coordinates.

(b) Convert the following equation of quadratic surface to standard form. What is this surface? 10

$$4x^2+y^2+4z^2-16x-2y+17=4$$

Q.No.8. (a) Find curvature of the space curve 10

$$\vec{r}(t) = 2t\hat{i} + t^2\hat{j} + \frac{1}{3}t^3\hat{k}$$

(b) (1) Find first fundamental form of the surface 10 (6+4)

$$\vec{r}(u,v) = (\cos u, \sin u, v)$$

(2) Write formulae for normal and Guassian curvature of a surface

$$\vec{r} = \vec{r}(u,v)$$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
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Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II. ALL** questions carry **EQUAL** marks.
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(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(vii) **Use of Calculator is allowed.**

SECTION-I

Q. No. 1. (a) Use the Mean Value Theorem to show that **(10)**
$|sinx - siny| \leq |x - y|$
for any real number x and y.

(b) Use Taylor's Theorem to prove that **(10)**
$lnsin(x+h) = lnsinx + hcotx - \frac{1}{2}h^2csc^2x + \frac{1}{3}h^3cotxcsc^2x + \cdots.$

Q. No. 2. (a) Evaluate $\lim_{x\to 0} \frac{sinx - \ln(e^x cosx)}{xsinx}$. **(8)**

(b) Find the equation of the asymptotes of $2xy = x^2 + 3$. **(6)**

(c) Evaluate the integral $\int_0^2 x^3(\sqrt{2x+3})dx$. **(6)**

Q. No. 3. (a) Verify that $f_{xy} = f_{yx}$ for the following function: **(8)**
$f(x,y) = e^{xy}\cos(bx+c)$.

(b) Find the points of relative extrema for $f(x) = sinxcos2x$. **(6)**

(c) Evaluate the limit $\lim_{x\to 0} \frac{1-cosx}{x^2}$. **(6)**

Q. No. 4. (a) Let $d: X \times X \to R$ be a metric space. Then $d': X \times X \to R$ defined by **(10)**
$d'(x,y) = \frac{d(x,y)}{1+d(x,y)}$
is also a metric.

(b) Show that an open ball in metric space X is an open set. **(5)**

(c) Show that convergent sequence in a metric space is Cauchy sequence. **(5)**

Q. No. 5. (a) Let (X,d) be a metric space, a subset A of X is dense if and only if A has non-empty intersection with any open subset of X. **(8)**

(b) Determine whether the given series converges or diverges:
$\sum_1^\infty \frac{(2n)!}{4^n}$ **(6)**

(c) Determine whether the given series converges absolutely, converges conditionally or diverges:
$\sum_1^\infty \frac{(-1)^n n!}{(2n)!}$. **(6)**

PURE MATHEMATICS, PAPER-II

SECTION-II

Q. No. 6. (a) Use De Moivre's Theorem to evaluate $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$. **(10)**

(b) Evaluate $\oint_C \frac{z+2}{z} dz$, where C is the circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq 2\pi)$. **(10)**

Q. No. 7. (a) Find the Laurent series that represents the function: **(10)**

$$f(z) = z^2 sin\left(\frac{1}{z^2}\right).$$

(b) Evaluate the sum of the infinite series: **(10)**

$$cos\theta - \frac{1}{2}cos2\theta + \frac{1}{3}cos3\theta - \frac{1}{4}cos4\theta + \cdots.$$

Q. No. 8. (a) Find the Fourier transform of : **(10)**

(i) $f(t) = e^{-|t|}$ (ii) $f(t) = sin\alpha t^2$

(b) Find the residue at $z = 0$ of the functions: **(10)**

(i) $f(z) = \frac{1}{z+z^2}$ (ii) $f(z) = zcos\left(\frac{1}{z}\right)$

FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2016
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Prove that the normaliser of a subset of a group G is a Subgroup of G. (10)

(b) Let A be a normal subgroup and B a subgroup of a group G. Then prove that $<A,B> = AB$ (10) **(20)**

Q. 2. **(a)** Let a be a fixed point of a group G and consider the mapping $I_a : G \rightarrow G$ defined by $I_a(g) = aga^{-1}$ where $g \in G$. (10)

Show that I_a is an automorphism of G. Also show that for $a, b \in G$, $I_a . I_b = I_{ab}$ (10) **(20)**

(b) Let $M_2(R) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R \right\}$ be the set of all 2×2 matrices with real entries. Show that ($M_2(R)$, +, ·) forms a ring with identity. Is ($M_2(R)$, +, ·) a field?

Q. 3. **(a)** Let $T: X \rightarrow Y$ be a linear transformation from a vector space X into a Vector space Y. Prove that Kernal of T is a subspace. (10)

(b) Find the value of λ such that the system of equations (10) **(20)**

$$x + \lambda y + 3z = 0$$
$$4x + 3y + \lambda z = 0$$
$$2x + y + 2z = 0$$

has non-trivial solution.

SECTION-B

Q. 4. **(a)** Using δ - $\in$ definition of continuity, prove that the function $Sin^2 x$ is continuous for all $x \in$ R. (10)

(b) Find the asymptotes of the curve $(x^2-y^2)(x+2y) + 5(x^2+y^2) + x+y = 0$ (10) **(20)**

Q. 5. **(a)** Prove that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$ (10)

PURE MATHEMATICS

Q. 6. **(a)** Find the area enclosed between the curves $y=x^3$ and $y=x$. (10)

(b) A plane passes through a fixed point (a, b, c) and cuts the coordinate axes in A, B, C. Find the locus of the centre of the sphere OABC for different positions of the plane, O is the origin. (10) **(20)**

SECTION-C

Q. 7. **(a)** Determine $P(z)$ where (10)

$P(z)=(z-z_1)(z-z_2)(z-z_3)(z-z_4)$ with $z_1=e^{i\pi/4}, z_2=\bar{z}_1, z_3=-z_1$ and $z_4=-\bar{z}_1$.

(b) Find value of the integral $\int_c (z-z_0)^n \, dz$, (n any integer) along the circle C (10) **(20)**

..........with centre and z_0 radius r, described in the counter clock wise direction.

Q. 8. **(a)** Use Cauchy Integral Formula to evaluate $\int_c \frac{\cos hz + \sin 2z}{z - \pi/2} dz$ along the simple closed counter C: $|z|=3$ described in the positive direction. (10)

(b) State and prove Cauchy Residue Theorem. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let H, K be subgroups of a group G. Prove that HK is a subgroup of G if and only if $HK=KH$. (10)

(b) If N, M are normal subgroups of a group G, prove that (10) **(20)**

$$NM/M \cong N/N \cap M.$$

Q. 2. **(a)** If R is a commutative ring with unit element and M is an ideal of R then show that M is a maximal ideal of R if and only if R/M is a field. (10)

(b) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then show that we can find elements a and b in F such that (10) **(20)**

$$1 + \alpha a^2 + \beta b^2 = 0.$$

Q. 3. **(a)** Let V be a finite-dimensional vector space over a field F and W be a subspace of V. Then show that W is finite-dimensional, (10)

$$\dim W \leq \dim V \text{ and } \dim V/W = \dim V - \dim W.$$

(b) Suppose V is a finite-dimensional vector space over a field F. Prove that a linear transformation $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. (10) **(20)**

SECTION-B

Q. 4. **(a)** Use the Mean-Value Theorem to show that if f is differentiable on an interval I, and if $|f'(x)| \leq M$ for all values of x in I, then (10)

$$|f(x) - f(y)| \leq M|x - y|$$

for all values of x and y in I. Use this result to show further that

$$|\sin x - \sin y| \leq |x - y|.$$

(b) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable at t, and if $z = f(x, y)$ is differentiable at the point $(x, y) = \big(x(t), y(t)\big)$, then $z = f\big(x(t), y(t)\big)$ is differentiable at t and (10) **(20)**

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y).

Q. 5. **(a)** Evaluate the double integral (10)

$$\iint_R (3x - 2y)\, dx\, dy$$

(b) Where R is a region enclosed by the circle $x^2 + y^2 = 1$. (10) **(20)**

Find the area of the region enclosed by the curves

$$y = \sin x, \quad y = \cos x, x = 0, \quad x = 2\pi.$$

Q. 6. **(a)** Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)

(b) Show that if a, b and c are nonzero, then the plane whose intercepts with the coordinate axes are $x = a$, $y = b$, and $z = c$ is given by the equation. (10) **(20)**

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

SECTION-C

Q. 7. **(a)** Prove that a necessary and sufficient condition that (10)

$$w = f(z) = u(x, y) + iv(x, y)$$

be analytic in a region R is that the Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{and} \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

are satisfied in R where it is supposed that these partial derivatives are continuous in R.

(b) Show that the function $f(z) = \bar{z}$ is not analytic anywhere in the complex plane Z. (10) **(20)**

Q. 8. **(a)** Let $f(z)$ be analytic inside and on the boundary C of a simply-connected region R. Prove that (10)

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz.$$

(b) Show that (10) **(20)**

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2} = \frac{5\pi}{32}.$$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2018
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE:
(i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
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SECTION-A

Q. 1. **(a)** Let H and K be normal subgroups of a group G. Show that HK is a normal subgroup of G. (10)

(b) Let H and K be normal subgroups of a group G such that $H \subseteq K$. Then show that (10) **(20)**

$$\frac{(G/H)}{(K/H)} \cong G/K$$

Q. 2. **(a)** Show that every finite integral domain is a field. (10)

(b) Consider the following linear system, (10) **(20)**

$$x + 2y + z = 3$$
$$ay + 5z = 10$$
$$2x + 7y + az = b$$

(i) Find the values of a for which the system has unique solution.

(ii) Find the values of the pair (a, b) for which the system has more than one solution.

Q. 3. **(a)** Find condition on a,b,c so that vector (a,b,c) in $\mathbb{R}^3$ belongs to (10)

$W = span\,\{u_1,u_2,u_3\}$ where $u_1 = (1,2,0)$, $u_2 = (-1,1,2)$, $u_3 = (3,0,-4)$.

(b) Let W_1 and W_2 be finite dimensional subspaces of a vector space V. Show that (10) **(20)**

$$dimW_1 + dimW_2 = dim\,(W_1 \cap W_2) + dim\,(W_1 + W_2)$$

SECTION-B

Q. 4. **(a)** Let $f(x) = \begin{cases} x^2 & if \;\; x \le 1 \\ x & if \;\; x > 1 \end{cases}$ (10)

Does the Mean Value Theorem hold for f on $\left[\frac{1}{2}, 2\right]$.

(b) Calculate the. $\lim\limits_{x \to 0} \dfrac{lnsin3x}{lnsinx}$ (10) **(20)**

Q. 5. **(a)** Evaluate $\int_{-1}^{5} |x-2|\, dx$. (10)

(b) Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$ if (10) **(20)**

$$f(x,y) = \begin{cases} x^2 y \sin \dfrac{1}{x} & when\ x, y\ are\ not\ both\ 0 \\ 0 & when\ x, y\ are\ both\ 0 \end{cases}$$

Q. 6. (a) Find the area of the region bounded by the cycloid (10)

$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$ and its base.

(b) Find the equation of a plane through (5,-1,4) and perpendicular to each of the planes (10) **(20)**

$$x + y - 2z - 3 = 0 \text{ and } 2x - 3y + z = 0$$

SECTION-C

Q. 7. (a) Express $\cos^5\theta \sin^3\theta$ in a series of sines of multiples of θ. (10)

(b) Use Cauchy's Residue Theorem to evaluate the integral $\int_C \frac{5z-2}{Z(Z-1)} dz$ where C is the circle $|z| = 2$, described counter clock wise. (10) **(20)**

Q. 8. (a) Find the Laurent series that represent the function $f(z) = \frac{z+1}{z-1}$ in the domain $1 < |z| < \infty$. (10)

(b) Expand $f(x) = sinx$ in a Fourier cosine series in the interval $0 \le x \le \pi$. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2019
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Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
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SECTION-A

Q. 1. **(a)** Show that the order and the index of a subgroup divides the order of a finite group. (10)

(b) Show that every finite integral domain is a field. (10) **(20)**

Q. 2. **(a)** Show that the characteristic of an integral domain is R is either zero or a prime. (10)

(b) Determine whether or not the set $\{(1,2,-1),(0,3,1),(1,-5,\ 3)\}$ of vectors is a basis for R^3. (10) **(20)**

Q. 3. **(a)** Show that a one-to-one linear transformation preserves basis and dimension. (10)

(b) Solve the system of linear equations: (10) **(20)**

$$2x_1 + x_2 + 5x_3 = 4$$
$$3x_1 - 2x_2 + 2x_3 = 2$$
$$5x_1 - 8x_2 + 2x_3 = 1.$$

SECTION-B

Q. 4. **(a)** Solve $\int_0^{\frac{\pi}{2}} \sin^2 6x \cos^4 3x\, dx$. (10)

(b) Find the area enclosed by $y = \frac{6}{2-\cos\theta}$. (10) **(20)**

Q. 5. **(a)** Show that in any conic semi-latusrectum is the harmonic mean between the segments of focal chord. (10)

(b) Prove that the evolute of hyperbola (10) **(20)**

$$2xy = a \text{ is } (x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}.$$

Q. 6. **(a)** Define Supremum and Infimum of a sequence. Find the supremum and infimum of the set (10)

$$\left\{(-1)^n\left(1-\frac{1}{n}\right), n = 1,2,3\ldots\right\}.$$

(b) Evaluate (10) **(20)**

$$\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}.$$

SECTION-C

Q. 7. **(a)** Show that $Log(1 + \cos\theta + i\sin\theta) = \ln(2\cos\frac{\theta}{2}) + i\frac{\theta}{2}$. (10)

(b) Find v such that $f(z) = u + iv$ is analytic. (10) **(20)**

Q. 8. **(a)** Prove that the series $z(1-z) + z^2(1-z) + z^3(1-z) + \cdots$ converges for $|z| < 1$, and find its sum. (10)

(b) Find the residues of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2020
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let G and G' be two groups and $f : G \to G'$ be a homomorphism then prove the following: (10)

(i) $f(e) = e'$ where e and e' are the identities of G and G' respectively

(ii) $f(a^{-1}) = [f(\text{a})]^{-1}$, $\forall a \in G$

(b) Prove that every homomorphic image of a group is isomorphic to some quotient group. (10) **(20)**

Q. 2. **(a)** A ring R is without zero divisor if and only if the cancellation law hold. (10)

(b) Prove that arbitrary intersection of subrings is a subring. (10) **(20)**

Q. 3. **(a)** Let $\text{T} : \text{R}^3 \longrightarrow \text{R}^3$ be the linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$. Find a basis and dimension of Range of T. (10)

(b) Prove that every finitely generated vector space has a basis. (10) **(20)**

SECTION-B

Q. 4. **(a)** Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the open intervals on which f is increasing and on which f is decreasing. (10)

(b) Find the horizontal and vertical asymptotes of the graph of $f(x) = -\dfrac{8}{x^2 - 4}$ (10) **(20)**

Q. 5. **(a)** Calculate $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$. (10)

(b) Find $\dfrac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$ if $w = x^2 + y^2 + z^2$, $z^3 - xy + yz + y^3 = 1$ and x and y are the independent variables. (10) **(20)**

Q. 6. **(a)** Determine the focus, vertex and directrix of the parabola $x^2 + 6x - 8y + 17 = 0$ (10)

(b) Find polar coordinates of the point p whose rectangular coordinates are $\left(3\sqrt{2}, -3\sqrt{2}\right)$ (10) **(20)**

SECTION-C

Q. 7. **(a)** Show that $\left(\cos\theta + i\sin\theta\right)^n = \cos(n\theta) + i\sin(n\theta)$ for all integers *n*. (10)

(b) Find the *n*, *n*th roots of unity. (10) **(20)**

Q. 8. **(a)** Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$. Where, if anywhere, does the series converge to $\frac{1}{x}$? (10)

(b) Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, ($p$ a real constant) converges if $p > 1$, and diverges if $P < 1$ (10) **(20)**

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